Solution to Assignment 5

Supplementary Problems

1. Use Fubini's theorem to obtain the area formula for a parallelogram. You may assume the three vertices of the parallelogram are $(0,0),(a_1,a_2),(b_1,b_2)$, where $a_i,b_i,i=1,2$, are all positive.

   Solution. Take $b_1<a_1$ and $a_2<b_2$ as a typical case. The fourth vertex of this parallelogram is (a_1+a_2,b_1+b_2). We separate the integral for area into three:

$$\int_0^{a_1}\int_{a_2x/a_1}^{b_2x/b_1} dydx+\int_{b_1}^{a_1}\int_{a_2x/a_1}^{a_2x/a_1+b} dydx+\int_{a_1}^{a_1+a_2}\int_{b_2x/b_1+c}^{a_2/a_1x+b} dydx\ ,$$

   where $b=(a_1b_2-a_2b_1)/a_1$ and $c=(a_2b_1-a_1b_2)/b_1$. Perform the integration to get the area formula: $|a_1b_2-a_2b_1|$.

2. Let

$$F(t)=\iiint_\Omega f(x^2+y^2+z^2)\,dV\ ,$$

   where Ω is the ball of radius t centered at the origin and f is continuous.

   Solution. In spherical coordinates,

$$F(t)=\int_0^{2\pi}\int_0^{\pi}\int_0^t f(\rho^2)\rho^2\sin\varphi\,d\rho d\varphi d\theta\ .$$

   Therefore,

$$F(t)=2\pi\times 2\times\int_0^t f(\rho^2)\rho^2\,d\rho\ ,$$

   and

$$F'(t)=4\pi t^2 f(t^2)\ .$$